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A NEW SAMPLING METHOD FOR STONE CRUSHING CIRCUITS

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BY

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SUMMARY

This report contains the results of samplings carried out during the month of October 1984 at the LAVAL quarry of the DEMIX company. The purpose of the samplings was to verify the accuracy and effectiveness of a method that would make it possible to obtain samples directly from a conveyor belt in operation.

This method makes it possible:

- To obtain better control of the quality of the finished products.
- To monitor and adjust the operational characteristics of crushing and screening units in full production, in order to obtain optimal performance.
- To minimize the amounts of material required and the number of times a samples must be handled to obtain valid results;

without production shutdowns and at a very reasonable cost.

First, we will examine the main sampling methods used today, in order to emphasize their various weaknesses (production shutdowns, weight of the samples). Then, we will explain and describe a sampling method that makes it possible to eliminate most problems encountered with currently used methods.

INTRODUCTION

This sampling method was developed within the context of a project leading to the development of a crushing plant simulator. However, this work required a preliminary series of samplings that would generate basic data in respect to the size distribution of the various types of materials flowing through the plant, in order to obtain a close fit between the theory and reality.

Thus, we had to choose a sampling method that would allow us to obtain samples from all the important points in the line. However, the techniques normally used in quarries had certain disadvantages which, as we will see below, made it difficult, or even impossible, to use them to obtain a thorough sampling of the materials in a crushing plant.

For these reasons, we had attempted to develop a flexible and inexpensive method that can be used to eliminate the disadvantages of conventional techniques, and make it possible to obtain representative samples. To this end, our first step was to verify and establish the effectiveness and precision of this method on the basis of recognized standards.

1- CONVENTIONAL SAMPLING METHODS

A. Sampling with the Conveyor at a Standstill

This method consists of collecting all the material over the entire cross-section of the flow present on the conveyor belt when it is at a standstill. The width of the section is determined on the basis of certain standards. The use of this method is the only way to ensure that all the particles are collected and that the sample is free of systematic errors. Thus, it remains the simplest and most precise method. However, with this method, the conveyor must be stopped for each sampling and, depending upon the location of the conveyor in the line, this also frequently involves the shutdown of many other conveyors, or even the entire quarry. Thus, when several samples must be obtained, this method causes partial, if not total,

and frequent production shut-downs, apart from the fact that it is sometimes impossible to re-start a loaded conveyor.

B. Sampling at discharge point

This method consists of passing a container through the flow of material in free fall. The container must be designed in such a way as to ensure that the material is collected through the entire width and thickness of the flow. This is the best method to obtain satisfactory samples when the material is in movement. However, because of their location, it is often physically impossible to obtain samples when dumping a conveyor or screening machine. Moreover, this method requires a sampling device that must be built as a function of each individual instaliation, and this may be quite costly.

C- Sampling in the Stockpile (Reserves)

In this case, the samples are taken directly from stockpiles of the product. It requires no production shutdowns, but it is much less precise than the previous methods, due to the possible segregation of the material in the piles. Moreover, it makes it possible to obtain samples only at the end of the line, which means that only the size distribution of the products can be determined.

2- PROPOSED METHOD

A- Characteristics

As we have seen, all conventional methods have certain disadvantages which render them incompatible with our intended use (see Introduction).

Thus, in order to solve these problems, our proposed sampling technique must have the following characteristics:

- It must make it possible to obtain one or more samples without interrupting production at any time.

- It must make it possible to obtain representative samples, which are not affected by systematic errors.

- It must make it possible to collect data at all points in the line (crushers, screens, separation, recombination, etc.)

- It must be inexpensive, simple, and flexible.

After examining the previously described characteristics, we reached the conclusion that sampling with the conveyor in operation is the only method that can meet these requirements. Thus, the samples must be obtained using a scoop, as will be described below.

With this method, it is not necessary to interrupt production, and samples can be obtained at all accessible point in the line. Moreover, themethod is simple and inexpensive.

B- Description of the Method and Its Components

The system developed includes two principal components: the scoop itself, and a framework, fixed to the conveyor, to which the scoop is attached.

The sampling scoop used has already been used to obtain samples. However, it is not used in exactly the same way. Its construction is simple, but must meet the requirements of the standards established by the International Standards Organization, as far as its dimensions are concerned (Fig. 1). Our model meets these standards, but we have modified the handle of the scoop, in order to make it possible to attach it to the framework (Fig. 2).

Since the risk of accident is always present in the vicinity of a working conveyor, we decided to attach the scoop to the framework in order to facilitate handling. Thus, its weight and that of the sample are entirely carried by the framework. The operator only has to lower the scoop into the material on the conveyor and then lift it using the handle as a lever arm. Moreover, the use of this framework makes it possible to maintain a certain consistency during the process of obtaining the data.

The framework consists of 2 uprights fixed to the sides of the conveyor. The scoop has a handle with two holes, and is placed on a pipe, which acts as a pivot and is itself held by the framework.

Once the scoop is in place, it is directly in front of the material. The sample is obtained by lowering the scoop into the material until it comes in contact with the belt, in order to collect all the fines. Once it is full, the scoop is brought back to its original position (Fig. 3). Care should be taken to ensure that the scoop does not remain too long into the material, in order to prevent overflows.

The position of the scoop on the framework must be chosen in such a way that the angle at which the scoop enters the material is neither too large nor too small. If the angle is too large there will be problems during the process of filling and lifting the scoop; and if the angle is too small, this can result in filling the scoop too quickly, so that it is full before reaching the bottom of the conveyor.

3- TEST

A- Reference Method

At this point, we must determined whether or not this method makes it possible to obtain adequate and representative samples. As we have already said, the method which makes it possible to obtain samples from a section of a conveyor at a standstill is the only way to ensure that all particles are collected, and consequently, that the sample is free of systematic errors. For this reason, we have used this method as the reference method. By comparing the results obtained with this method and those obtained with the scoop, we will be able to determine whether the latter can be used to obtain representative samples of the material.

B- Testing Procedure

In order to ensure a certain consistency in the process of collecting the data, we established the following procedure: samples are obtained at regular intervals using the scoop while the conveyor is in operation. After

the samples is obtained, the conveyor is immediately stopped. Then, a new sample is obtained using the reference method. Moreover, the sample must be obtained as close as possible to the place where the sample obtained with the scoop was taken. In this way, we minimize the risk that the two samples may have been affected by fluctuations in the material, and consequently, the two methods should theoretically produce identical results in a size distribution analysis.

C- Parameters

In order to carry out the tests, it is first necessary to determine the following parameters:

- Location of the sampling
- Maximum grain size of the product to be samples
- Minimum required weight of the sample
- Minimum size of the scoop
- Length of the section
- Required number of samples.

Sample Location

The conveyor used to obtain the reference sample must be chosen in such a way that its shutdown affects products as little as possible.

After studying the flow chart of the plant (Fig. 4), we chose to sample the product when it was 0-20 mm (conveyor C-33). This conveyor was located at the beginning of the line, so that shutting it down required stopping only one other conveyor (C-32b).

Maximum Grain Size of the Product to be Samples

The maximum grain size determines the size of the openings of a screen which can regain about 5% of the material (ISO Standard 3081). This value serves to determine the size of the sampling scoop as well as the section of the conveyor to be sampled. It is also used to determine the minimum weight of the sample. In our case, the maximum grain size of the material was about 20 mm.

Minimum Required Weight of the Sample

As we have said, the maximum grain size is used, among other things, to determine the minimum weight that must be obtained for each sample. By obtaining samples of this minimum weight, we are reasonably sure that we are getting a representative sample.

During the course of a normal sampling process, a given number of elementary samples are obtained and then these are recombined in order to form a total sample. Then, this sample is divided in the laboratory, in order to obtain a definitive sample which will be subjected to the granulometric analysis. However, in our case, in order to meet the requirements of the test, we analyzed each elementary sample separately. This process is the recommended procedure for better precision.

However, it is necessary to meet the standards as far as the minimum weight of each elementary sample and the total weight of the sample are concerned, even if the samples are not to be recombined.

ASTM Standard D-75 recommends a minimum weight of 35 kg for a total sample, when the maximum grain size of the material is 20 mm and consists of both coarse and fine particles (Table 2).

ISO Standard 3081 also requires a minimum weight of 0.8 kg for each elementary sample when the material has a maximum grain size of 20 mm (Table 3).

Minimum Size of the Scoop

Again, the size of the scoop is a function of the maximum grain size of the material. ISO Standard 3081 indicates that for a maximum grain size of 20 mm, it is necessary to use a Number 20 scoop (Table 1). Since this is a minimum requirement, we decided to use a Number 50 scoop, in order to obtain samples of the valid weight. Thus, the samples obtained weighed about 4 kg.

Length of the Section

In accordance with ISO Standard 3081, the length of the section of the conveyor over which the sample is collected must be determined in such a way

that the weight of the sample meets the value specified in the section entitled "Minimum Required Weight of the Sample", and it must also be at least three times the maximum grain size, that is, 60 mm.

In order to facilitate the sampling process, and only for this reason, we chose a length of 30 cm. This should produce samples weighing between 5 kg and 8 kg, depending upon the thickness of the material on the conveyor.

Required Number of Samples

The number of samples to be obtained must be determined in such a way that the sample is as representative as possible of a product with a grain size of 0-20 mm.

A sampling consultant [1] recommended that we obtain between 10 and 20 samples, in order to make it possible to reach valid conclusions regarding the effectiveness of the method.

Thus, we decided to obtain 40 samples [2] over two (2) consecutive days.

4- METHOD USED FOR THE SIZE DISTRIBUTION ANALYSIS OF THE SAMPLES

The size distribution analyses of the samples were carried out at the DEMIX Laboratories in LAVAL.

Each sample was first screened until we obtained a grain size of 4.75 mm. The residue was then divided in order to obtain sub-samples weighing about 500 g. These samples were then wet screened for grain sizes below 4.75 mm.

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(1) MATRIX CONSULTANTS LIMITED, Mr. Jan W. Merks.

(2) Including 20 reference samples

5- RESULTS AND CALCULATIONS

A- Compilation of the Results

The samples were divided into four groups depending upon the sampling method used and the day when they were obtained.

GROUP 1:	SS* 1 to SS 10	Conveyor section 30/10/84
GROUP 2:	SS 11 to SS 20	Conveyor section 31/10/84
GROUP 3:	MS** 1 to MS 10	Sampling scoop 30/10/84
GROUP 4:	MS 11 to MS 20	Sampling scoop 31/10/84

The results of the size distribution analyses of the samples are shown in Tables 4, 5, 6 and 7.

B- Mean, Variance, Standard Deviation

For each size distribution class of each of the groups, we calculated the mean (X), variance (S^2), and standard deviation S), using the following equations:

$$X = \sum_{i=1}^{n} \frac{xi}{n}$$

$$S^2 = \sum_{i=1}^{n} \frac{(xi - X)^2}{n - 1}$$

$$S = \sqrt{S^2}$$

where n = number of samples

The results of these calculations are shown in Tables 8 to 11.

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* SS: Section at a standstill

**MS: Moving section

C- Confidence Intervals

Since the sample was obtained from a normal population of unknown variance, and the size of the sample was under ($n < 30$), we used Student's law as the distribution model.

In our case, this law is applicable, and it made it possible, among other things, to determine an interval with a confidence level of 1 - a, where the limits are:

$$LI = \overline{X} - (t\ a/2;\ n-1) \times \frac{S}{\sqrt{n}}$$

$$LS = \overline{X} + (t\ a/2;\ n-1) \times \frac{S}{\sqrt{n}}$$

where: a = confidence threshold
n-1 = degrees of freedom
t a/2; n-1 = Student's factor
LI = lower limit
LS = upper limit

Depending upon the confidence level chosen, this interval has the probability of including the real mean (M) of a given population.

$$P\ (LI \leq M \leq LS) = 1 - a$$

To obtain a confidence interval of 98% (a = 2%), we calculated the limits for each size distribution in the four groups. The results are shown in Tables 12 and 13.

D- Test on the Means

Our first verification consisted of comparing the means of two groups using a statistical test, in order to verify the equality of these means.

The test used is described in Section 7.20 of Baillargeon and Rainville's Applied Statistics. It consists of verifying the following hypothesis (HO):

$$HO : M1 = M2$$

where M1 and M2 are the hypothetical means of the groups being compared.

Depending upon the confidence level chosen, the calculation of the critical ratio will make it possible to accept or reject HO.

Since the number of samples is small (n1 and n2 < 30); the population is distributed normally; and the real variances of the group are unknown but assumed to be equal, since they were obtained form the same population; we can express the critical ratio t as follows:

$$t = \frac{(\bar{X}1 - \bar{X}2) - (M1 - M2)}{\sqrt{\frac{(n1 - 1)\, S1^2 + (n2 - 1)\, S2^2}{n1 + n2 - 2}} \times \sqrt{\frac{1}{n1} + \frac{1}{n2}}}$$

The critical ratio t is distributed in accordance to Student's law with n1 + n2 - 2 degrees of freedom.

where: X1, X2 = calculated means for each group
$S1^2$, $S2^2$ = calculated variances for each group
n1, n2 = number of samples in each group

In our case, n1 and n2 are the same, and since M1 and M2 are assumed to be equal on the basis of hypothesis HO, we can rewrite the equation as follows:

$$t = \frac{(X1 - X2)}{\sqrt{\frac{S1^2 + S2^2}{n}}}$$

where n = the number of samples in each group.

Finally, the decision criterion will be as follows:

Reject HO if $t > t_{a/2;\ 2n-2;}$
otherwise accept HO

We carried out this test by comparing groups 1 and 3 and groups 2 and 4 for a confidence level of 98% (a = 2%). The results are shown in Table 14.

E- Paired Observations - Test on the Differences

The second test that we carried out, and this is the most important, made it possible to detect the presence of systematic errors, and their size.

The test was carried out on the mean differences and is described in Appendix E of ISO Standard 1988, and Section 7.22 of Baillargeon and Rainville's Applied Statistics.

The principle is as follows: once the elementary samples have been obtained using both methods, the results of the size distribution analysis of each corresponding sample are then grouped in pairs (SS 1 and MS 1, SS 2 and MS 2, etc.). Then, on the basis of certain calaulations, we determine whether or not here is a significant difference between the results obtained using the two methods.

In order to do this, the first step is to calculate the difference, taking into account the sign, between the two members of each pair. We assume that this difference is a random normally distributed variable, with a mean Md and unknown variance Vd. Thus, the test is carried out on the mean difference d:

$$d = \frac{\sum_{i=n}^{n} di}{n}$$

where n = number of samplings

d = difference between the members of each pair

The standard error (standard deviation) Sd is:

$$Sd = \frac{Sd}{\sqrt{n}} \quad \text{or} \quad Sd = \sqrt{\frac{\sum_{i=1}^{n} (di - d)^2}{n - 1}}$$

Since this is a statistical test, the hypothesis that we must prove is as follows:

$$Ho : Md = 0$$

In other words, we want to prove that the mean of the differences equals 0, which implies that sampling with the scoop is not affected by systematic errors.

The critical ratio that serves to prove hypothesis Ho is:

$$t = \frac{d - Md}{Sd} = \frac{d}{Sd}$$

Since Md is assumed to equal 0.

The distribution follows Student's law with (n - 1) degrees of freedom. Thus, the decision criterion will be:

"Reject Ho if t > t a/2; n-1; otherwise accept Ho"

In more precise terms, d represents the systematic error and it is not significant if t < a/2; n-1.

We carried out a test for each size distribution interval by pairing the results of groups 1 - 3 and 2 - 4. The level of confidence was 98% (a=2%). The results are shown in Tables 15 to 18.

6- ANALYSIS

A- General

In this chapter we will analyze the data collected following the process described in the previous chapter. This analysis will make it possible to arrive at conclusions regarding the use of the scoop as an effective sampling method.

The tests are carried out by comparing the samples obtained with the scoop with those obtained over a section of the conveyor at a standstill, with the latter representing the reference method. Thus, we compared the samples obtained during the first day (Groups 1 and 3) and those obtained during the second day (Groups 2 and 4). Moreover, since we could not calculate a single representative characteristics for each sample, we had to carry out the test for each size distribution interval.

All the tests were carried out with an error level of 2% (confidence level of 98%). This means that we had a 2% chance of rejecting the hypothesis formulated for the test, when the hypothesis was true.

B- Test on the Means

The samples obtained with the scoop may lead to errors which can cause significant differences between the means of the groups of samples tested even if there are no differences between these groups, all the samples having been obtained from the same material. The test on the means, which was described in Section D of the previous chapter, makes it possible to determine whether there are any significant differences between the means.

By examining the results of the test (Table 14), we can reach the following conclusions:

The critical ratio is under Student's t for all size difference classes for both tests (groups 1 and 3, groups 2 and 4). Thus, the hypothesis of the equality of the means is verified in all cases.

The values of the critical ratio for the tests on groups 1 and 3 are much closer to zero than those of the tests carried out on groups 2 and 4. Thus, we conclude that the deviations between the means of the groups tested were lower the first day than the second.

Even though, as we said before, the deviations are larger for the second day, the tests show, that for a confidence level of 98%, the two methods produced samples where the mean size differences did not show significant differences.

Thus, since we know that the reference method is free of systematic errors, the first test confirms that obtaining samples with the scoop will not produce errors which would cause a significant difference between the mean of these samples and the real mean of the material.

C- Test on Paired Observations

The errors discussed in the previous section can also mask a significant difference between the means. Thus, the sampling scoop may produce, on the average, the same results as the reference method, but the results may differ greatly for individual measures. In order to verify this phenomenon, we used the test on paired observations (Section 5-E) in order not to calculate total performance, but rather to obtain a measurement of the deviations for each pair of samples.

In this way, we will truly be able to determine whether the method tested has a tendency to introduce systematic errors caused by the fact that the sample contains too many or too few coarse or fine particles.

By examining the results of this test (Table 17 and 18), we were able to establish the following points:

The critical ratio is under Student's t for all size distribution classes and for both tests (groups 1 and 3, groups 2 and 4). Thus, the hypothesis according to which mean difference is equal to zero is verified in all cases.

Thus, we are able to say, with a confidence level of 98%, that the sampling scoop makes it possible to obtain samples which are significantly free of systematic errors.

7- DISCUSSION

Thus, in the previous section, we were able to show that a sample obtained with the scoop is as representative and free of error as a sample obtained over a section of a conveyor at a standstill. However, as in the case of all other sampling methods, this particular method must also be easy to use and must meet certain standards. These standards are mainly those described in Section 3. On the other hand, the observations made during these tests, made it possible for us to establish other criteria that must be met when using the scoop sampling method.

The first point concerns the flow rate of the material on the conveyor. During the second day, we observed that the flow rate of the material had a tendency to change. For this reason, and in order to obtain samples of valid weight, we had to keep the scoop in contact with the material for a longer period of time, which tended to overfill the scoop and may have caused some segregation in the sample. The results of the second day tend to confirm this observation. Thus, it seems that the flow rate, or more specifically the thickness of the layer of material on the conveyor, must be enough to fill the scoop within a very short period of time, in order to prevent overfilling. This minimum thickness depends mainly upon the size of the scoop and the angle at which the scoop approaches the conveyor during the sampling process. Thus, before obtaining a sample, it is important to carry out a few preliminary tests with the scoop in different positions, in order to determine the optimal sampling angle. As a general rule, we can say that the angle of approach of the scoop should decrease, as the thickness of the layer of the material increases.

We carried out this test by obtaining two series of 10 samples with each method. However, during normal sampling, the number of samples required to ensure reasonable precision may differ. ASTM Standards D-2234,

E-105, E-122, and E-141 explain the procedures that must be used to determine the minimum number of samples required to obtain results with the desired level of confidence.

A second point refers to the use of a framework to support the scoop. Even though the scoop may be used without the framework, we consider that use of the latter is very important to maintain a certain consistency throughout the sampling process. The use of the framework makes it possible to do this, and must thus be considered an integral part of the system.

The test was carried out with a material with a maximum grain size of 20 mm. However, we believe that the scoop may produce satisfactory results with materials with a grain size of up to 100 mm. Due to the weight of the samples collected, the scoop and the framework had to be slightly modified, that is, reinforced, in order to facilitate handling.

Finally, it is important to emphasize that the location at which the sampling takes place must be determined carefully, One must ensure that there is no segregation of the material or any other phenomenon that may affect results at this particular point. Normally, obtaining the samples immediately after dumping or screening makes it possible to control for the effect of this phenomenon.

We can say that this sampling method produces more information about the performance of a crushing plant (without production shutdowns), and its finished products, than more conventional methods. It is important to emphasize that, in comparison to more conventional methods, this sampling method can lower costs and decrease the quantities required and the number of times the samples must be handled, while still producing valid results.

APPENDIX A

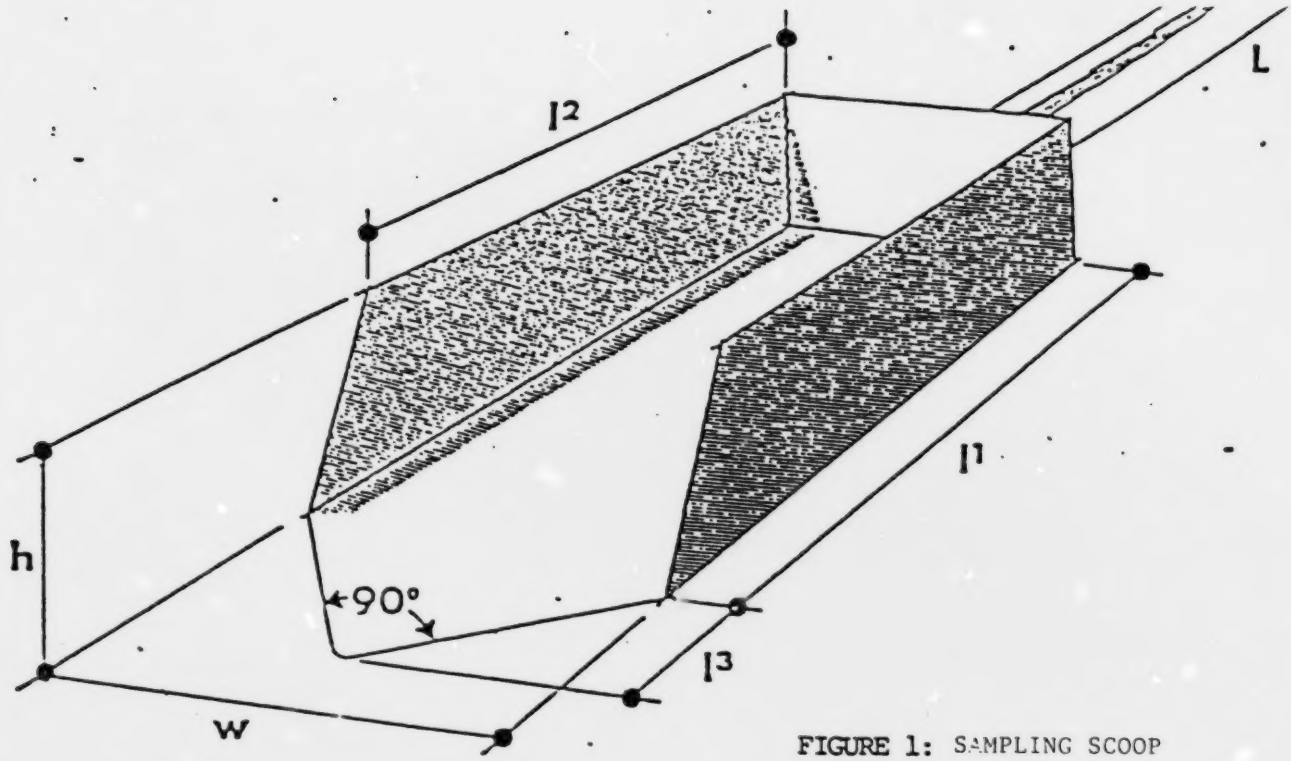


FIGURE 1: SAMPLING SCOOP

TABLE 1: DIMENSIONS OF THE SAMPLING SCOOP

SCOOP NO.	MAXIMUM GRAIN SIZE OF MATERIAL (mm)	VOLUME (ml)	DIMENSIONS (mm)				
			w	h	I1	I2	I3
1	1	15	30	15	30	25	12
3	3	40	40	25	40	30	15
5	5	75	50	30	50	40	20
10	10	125	60	35	60	50	25
15	15	200	70	40	70	60	30
20	20	300	80	45	80	70	35
30	30	400	90	50	90	80	40
40	40	800	110	65	110	95	50
50	50	1700	150	75	150	130	65
75	75	4000	200	100	200	170	80
100	100	7000	250	110	250	220	100
125	125	10000	300	120	300	250	120
150	150	16000	350	140	350	300	140

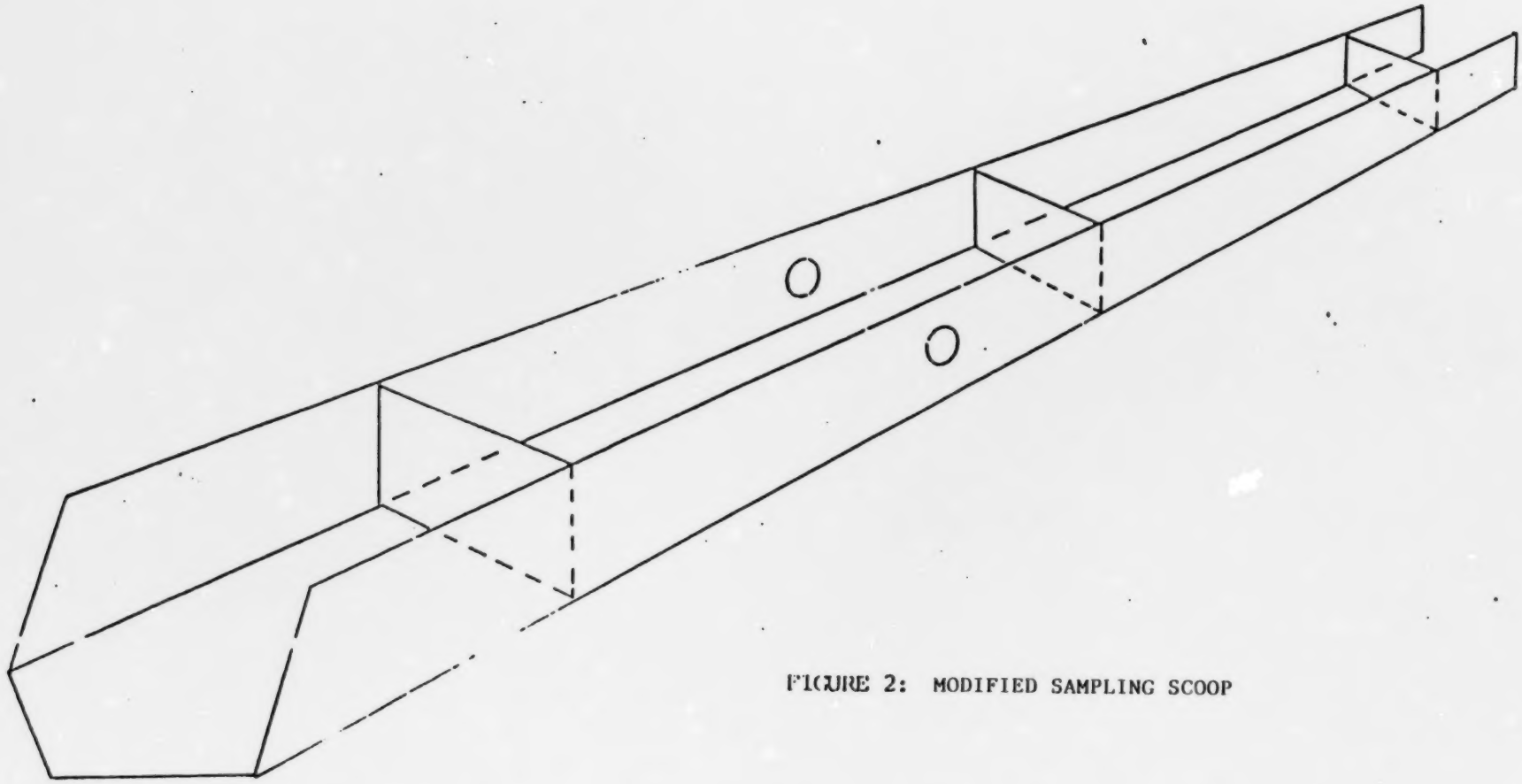


FIGURE 2: MODIFIED SAMPLING SCOOP

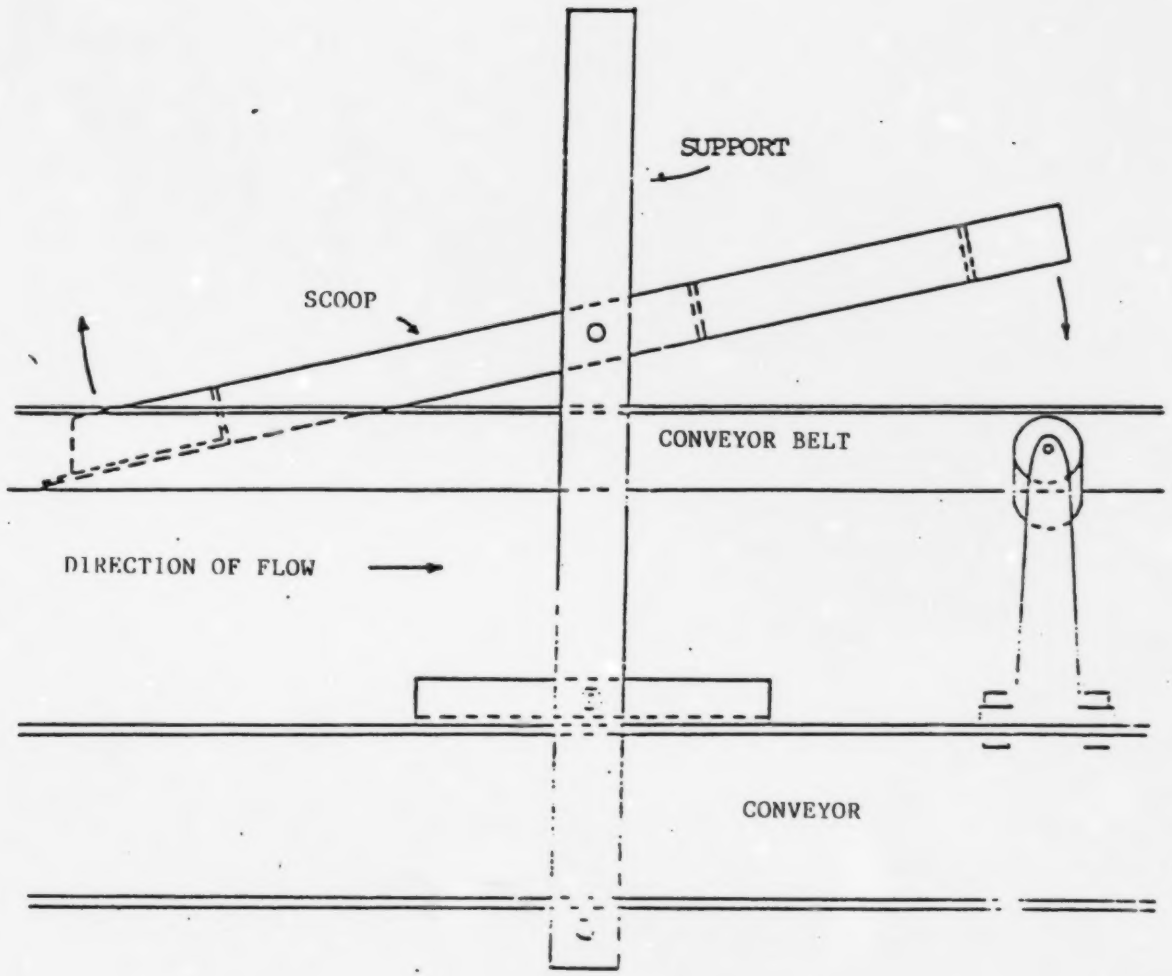
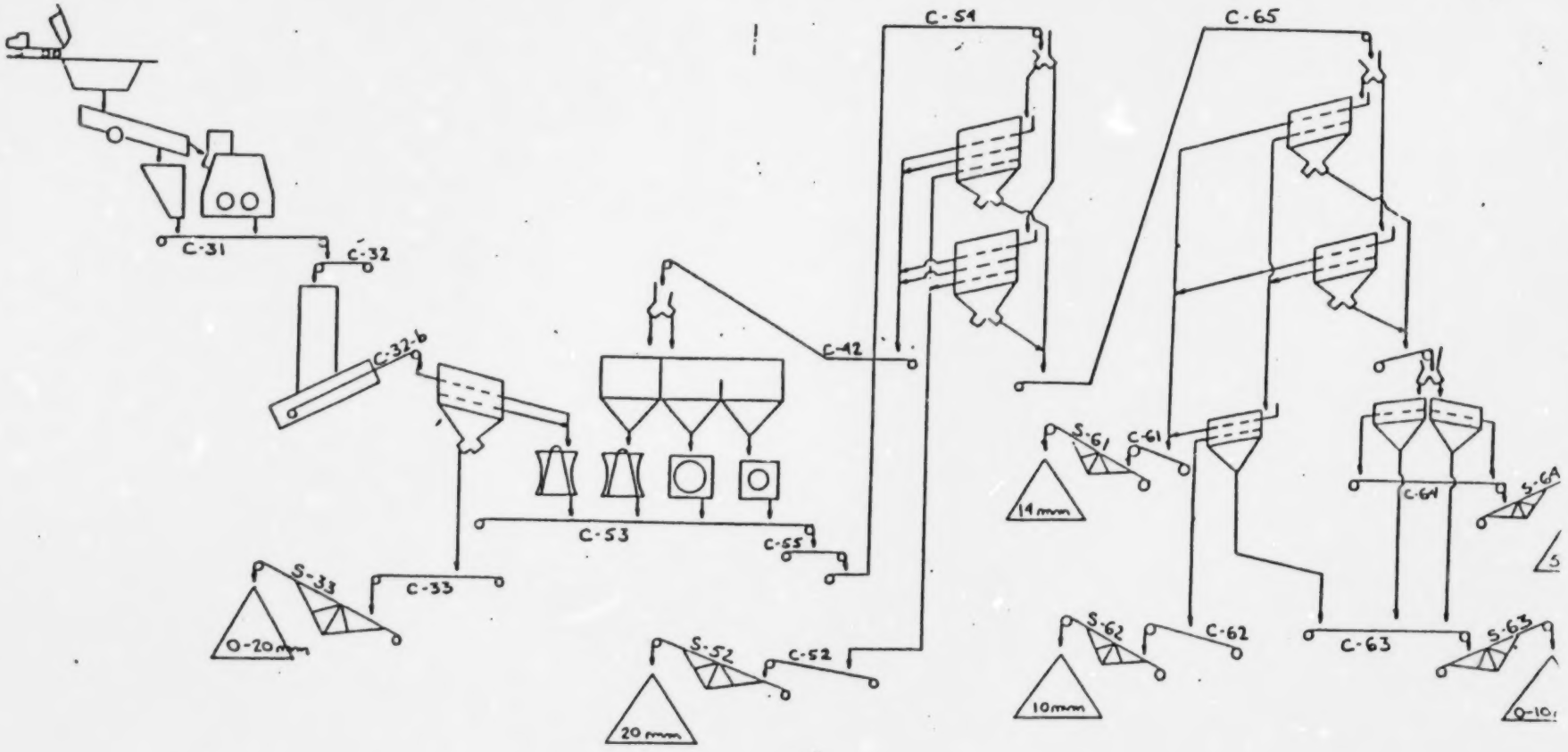


FIGURE 3: INSTALLATION OF THE SCOOP ON THE CONVEYOR



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DESSIN: DWG.	NO.	B-

TABLE 2: MINIMUM WEIGHT OF THE TOTAL SAMPLE

TYPE OF MATERIAL	MAXIMUM GRAIN SIZE OF THE PARTICLES (mm)	MINIMUM WEIGHT OF THE TOTAL SAMPLE (kg)*
FINE	2.5	10.0
	5.0	10.0
COARSE	10.0	10.0
	14.0	15.0
	20.0	25.0
	28.0	50.0
	40.0	75.0
	56.0	100.0
	80.0	150.0
	112.0	200.0

*In the case of a mixture of coarse and fine aggregates, add 10kg to the minimum recommended weight for the coarse aggregate.

TABLE 3: MINIMUM WEIGHT OF THE SAMPLE

MAXIMUM GRAIN SIZE OF THE PARTICLES (mm)		MINIMUM WEIGHT OF THE SAMPLES (kg)	SCOOP NUMBER
150	250	40	
100	150	20	
50	100	12	100
20	50	4	50
10	20	0.8	20
0	10	0.3	10

TABLE 4: RESULTS OF SIZE DISTRIBUTION ANALYSIS, GROUP I

DIM (microns)	SS-1	SS-2	SS-3	SS-4	SS-5	SS-6	SS-7	SS-8	SS-9	SS-10
25000	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
19000	4.3	2.7	3.5	2.6	1.9	4.7	2.7	1.8	2.0	1.9
12500	18.7	19.5	14.5	15.7	15.6	12.7	15.6	16.4	16.4	19.4
9500	11.8	12.3	11.4	10.7	11.5	10.0	11.2	9.9	11.3	11.6
4750	21.0	23.2	21.8	22.2	22.6	22.0	22.8	22.5	22.0	22.6
2360	13.9	14.4	15.1	14.6	15.8	15.3	14.3	14.0	15.3	14.1
1180	9.5	8.1	10.4	9.8	9.4	10.1	9.8	10.5	9.4	8.6
600	5.7	5.2	6.5	6.4	6.4	6.4	6.2	7.2	6.5	5.5
300	3.6	3.4	4.1	4.2	3.9	4.5	4.0	4.2	4.0	3.8
150	2.6	2.6	2.8	3.1	2.8	3.2	3.0	3.0	2.8	2.8
75	1.9	1.9	2.0	2.3	2.0	2.4	2.1	2.1	2.3	2.1
UNDERSIZE	7.0	6.7	7.9	8.4	8.1	8.7	8.3	8.4	8.0	7.6

VALUES IN RETAINED PERCENTAGE

TABLE 5: RESULTS OF SIZE DISTRIBUTION, GROUP 2

DIM (microns)	SS-11	SS-12	SS-13	SS-14	SS-15	SS-16	SS-17	SS-18	SS-19	SS-20
25000	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
19000	2.7	4.2	3.5	3.7	4.0	3.5	2.1	2.8	4.9	3.9
12500	21.3	25.7	23.6	18.8	17.9	21.6	18.0	15.3	17.5	22.4
9500	12.5	13.0	12.6	10.9	10.9	12.7	12.2	13.0	10.2	12.2
4750	22.7	20.7	20.8	22.0	21.5	22.0	24.4	24.1	21.2	20.9
2360	11.8	11.1	11.7	12.4	13.4	11.5	13.2	12.8	14.3	11.8
1180	7.9	7.1	7.3	8.9	8.8	8.1	9.1	9.3	9.0	8.1
600	4.9	4.3	5.1	5.8	6.1	5.1	5.3	5.8	6.2	5.0
300	3.1	2.7	3.2	3.7	3.8	3.3	3.3	3.5	3.7	3.2
150	2.4	2.1	2.5	2.8	2.8	2.4	2.4	2.6	2.6	2.4
75	2.1	1.7	2.1	2.2	2.3	2.0	2.0	2.1	2.1	1.9
UNDERSIZE	8.6	7.4	7.6	8.8	8.5	7.8	8.0	8.7	8.3	8.2

VALUES IN RETAINED PERCENTAGE

TABLE 6: RESULTS OF SIZE DISTRIBUTION, GROUP 3

DIM (microns)	MS-1	MS-2	MS-3	MS-4	MS-5	MS-6	MS-7	MS-8	MS-9	MS-10
25000	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
19000	4.4	2.7	3.4	1.9	3.4	2.3	1.9	2.2	2.4	1.6
12500	20.1	19.8	18.8	13.2	15.3	13.3	19.5	15.9	17.7	16.6
9500	12.3	10.1	10.5	8.7	11.5	11.5	11.8	10.1	12.5	12.2
4750	23.0	23.3	20.8	22.2	22.2	22.8	21.1	22.3	21.7	21.1
2360	12.6	15.3	13.9	15.7	14.4	15.4	14.1	15.5	14.0	13.5
1180	8.5	9.2	10.0	11.4	9.9	10.3	9.6	10.3	9.2	10.0
600	5.2	5.3	6.3	7.4	6.0	6.5	5.9	7.0	6.6	6.3
300	3.2	3.3	4.0	4.5	4.0	4.1	3.7	3.9	3.6	4.4
150	2.3	2.5	2.7	3.4	2.9	2.9	2.6	2.7	2.6	3.2
75	1.7	1.8	2.1	2.5	2.3	2.3	2.0	2.2	2.0	2.5
UNDERSIZE	6.7	6.7	7.5	9.1	8.1	8.6	7.8	7.9	7.7	8.6

VALUES IN RETAINED PERCENTAGE

TABLE 7: RESULTS OF SIZE DISTRIBUTION ANALYSIS, GROUP 4

DIM (microns)	MS-11	MS-12	MS-13	MS-14	MS-15	MS-16	MS-17	MS-18	MS-19	MS-20
25000	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
19000	3.0	3.1	1.8	1.8	1.7	1.6	5.4	4.7	1.0	1.8
12500	17.3	16.0	14.4	20.4	16.5	15.4	21.2	21.2	14.7	16.1
9500	12.3	14.2	11.2	13.1	10.6	12.8	12.7	10.6	10.0	13.3
4750	21.6	23.4	24.5	24.3	21.6	23.6	22.6	20.7	21.9	21.6
2360	13.1	13.7	13.3	12.4	15.2	14.1	11.4	12.6	15.5	13.7
1180	8.4	8.7	9.6	7.9	9.6	8.7	7.7	7.8	10.6	9.7
600	6.0	5.1	6.9	4.7	6.4	6.1	4.6	5.3	6.6	6.1
300	3.7	3.1	4.3	3.0	4.2	3.7	2.9	3.4	4.1	3.8
150	2.9	2.4	3.0	2.4	3.0	2.9	2.2	2.6	3.2	2.6
75	2.4	2.0	2.3	2.0	2.5	2.3	1.8	2.2	2.4	2.2
UNDERSIZE	9.3	8.3	8.7	8.0	8.7	8.8	7.5	8.9	10.0	9.1

VALUE IN RETAINED PERCENTAGE

TABLE 8: GROUP 1: MEAN, VARIANCE, STANDARD DEVIATION
(samples SS-1 to SS-10)

DIM (microns)	MEAN X	VARIANCE S^2	STANDARD DEVIATION S
19000	2.81	1.0743	1.0365
12500	16.45	4.7717	2.1844
9500	11.17	0.5823	0.7631
4750	22.27	0.3779	0.6147
2360	14.68	0.4262	0.6529
1180	9.56	0.5671	0.7531
600	6.20	0.3378	0.5812
300	3.97	0.1001	0.3164
150	2.87	0.0401	0.2003
75	2.11	0.0299	0.1729
UNDERSIZE	7.91	0.4099	0.6402

VALUES IN RETAINED PERCENTAGE

TABLE 9: GROUP 2: MEAN, VARIANCE, STANDARD DEVIATION
(samples SS-11 to SS-20)

DIM (microns)	MEAN X	VARIANCE S^2	STANDARD DEVIATION S
19000	3.53	0.6646	0.8152
12500	20.21	10.3566	3.2182
9500	12.02	0.9818	0.9908
4750	22.03	1.7646	1.3284
2360	12.40	1.0133	1.0066
1180	8.36	0.5982	0.7734
600	5.36	0.3604	0.6004
300	3.35	0.1117	0.3342
150	2.50	0.0444	0.2108
75	2.05	0.0272	0.1650
UNDERSIZE	8.19	0.2299	0.4795

VALUES IN RETAINED PERCENTAGE

TABLE 10: GROUP 3: MEAN, VARIANCE, STANDARD DEVIATION
(samples MS-1 to MS-10)

DIM (microns)	MEAN X	VARIANCE X^2	STANDARD DEVIATION S
19000	2.62	0.7551	0.8690
12500	17.02	6.6462	2.5780
9500	11.12	1.5040	1.2264
4750	22.05	0.7361	0.8580
2360	14.44	1.0271	1.0135
1180	9.84	0.6204	0.7877
600	6.25	0.4739	0.6884
300	3.87	0.1823	0.4270
150	2.78	0.1084	0.3293
75	2.14	0.0738	0.2716
UNDERSIZE	7.87	0.6157	0.7846

TABLE 11: GROUP 4: MEAN, VARIANCE, STANDARD DEVIATION
(samples MS-11 to MS-20)

DIM (microns)	MEAN X	VARIANCE S	STANDARD DEVIATION S
19000	2.59	2.1054	1.4510
12500	17.32	6.9529	2.6368
9500	12.08	1.9396	1.3927
4750	22.58	1.6929	1.3011
2360	13.50	1.5511	1.2454
1180	8.87	0.9423	0.9707
600	5.78	0.6462	0.8039
300	3.62	0.2551	0.5051
150	2.72	0.1062	0.3259
75	2.21	0.0477	0.2183
UNDERSIZE	8.73	0.4823	0.6945

TABLE 12: 98% CONFIDENCE INTERVAL, GROUPS 1 AND 2

	GROUP 1		GROUP 2	
DIMENSION (microns)	LI	LS	LI	LS
19000	1.88	3.73	2.80	4.26
12500	14.50	18.40	17.34	23.08
9500	10.49	11.85	11.14	12.90
4750	21.72	22.82	20.84	23.22
2360	14.10	15.26	11.50	13.30
1180	8.89	10.23	7.67	9.05
600	5.68	6.72	4.82	5.90
300	3.69	4.25	3.05	3.65
150	2.69	3.05	2.31	2.69
75	1.96	2.26	1.90	2.20
UNDERSIZE	7.34	8.48	7.76	8.62

*LI: Lower Limit

*LS: Upper Limit

TABLE 13: 98% CONFIDENCE INTERVAL, GROUPS 3 AND 4

	GROUP 3		GROUP 4	
DIMENSION (microns)	LI	LS	LI	LS
19000	1.84	3.40	1.30	3.88
12500	14.72	19.32	14.97	19.67
9500	10.03	12.21	10.84	13.32
4750	21.28	22.82	21.42	23.74
2360	13.54	15.34	12.39	14.61
1180	9.14	10.54	8.00	9.74
600	5.64	6.86	5.06	6.50
300	3.49	4.25	3.17	4.07
150	2.49	3.07	2.43	3.01
75	1.90	2.38	2.01	2.40
UNDERSIZE	7.17	8.57	8.11	9.35

*LI: Lower Limit

*LS: Upper Limit

TABLE 14: CRITICAL RATIO t FOR TEST ON THE MEANS

(a = 2%, t 0.01; 18 = 2.5524)

DIMENSION (microns)	Critical Ratio t	
	GROUPS 1-3	GROUPS 2-4
19000	0.4442	1.7860
12500	-0.5334	2.1966
9500	0.1095	-0.1110
4750	0.6591	-0.9354
2360	0.6295	-2.1722
1180	-0.8125	-1.2994
600	-0.1755	-1.3238
300	0.5951	-1.4098
150	0.7385	-1.7927
75	-0.2946	-1.8488
UNDERSIZE	0.1249	-2.0234

TABLE 15: DIFFERENCES, GROUPS 1-3

No of Samples	DIMENSIONS (microns) 19000	12500	9500	4750	2360	1180	600	300	150	75	-75
SS1-MS1	-0.1	-1.4	-0.5	-2.0	1.3	1.0	0.5	0.4	0.3	0.2	0.3
SS2-MS2	0.0	-0.3	2.2	-0.1	-0.9	-1.1	-0.1	0.1	0.1	0.1	0.0
SS3-MS3	0.1	-4.3	0.9	1.0	1.2	0.4	0.2	0.1	0.1	-0.1	0.4
SS4-MS4	0.7	2.5	2.0	0.0	-1.1	-1.6	-1.0	-0.3	-0.3	-0.2	-0.7
SS5-MS5	-1.5	0.3	0.0	0.4	1.4	-0.5	0.4	-0.1	-0.1	-0.3	0.0
SS6-MS6	2.4	-0.6	-1.5	-0.8	-0.1	-0.2	-0.1	0.4	0.3	0.1	0.1
SS7-MS7	0.8	-3.9	-0.6	1.7	0.2	0.2	0.3	0.3	0.4	0.1	0.5
SS8-MS8	-0.4	0.5	-0.2	0.2	-1.5	0.2	0.2	0.3	0.3	-0.1	0.5
SS9-MS9	-0.4	-1.3	-1.2	0.3	1.3	0.2	-0.1	0.4	0.2	0.3	0.3
SS10-MS10	0.3	2.8	-0.6	1.5	0.6	-1.4	-0.8	-0.6	-0.4	-0.4	-1.0
TOTAL	1.9	-5.7	0.5	2.2	2.4	-2.8	-0.5	1.0	0.9	-0.3	0.4

TABLE 16: DIFFERENCES, GROUPS 2-4

No. of Samples	DIMENSIONS (microns)										
	19000	12500	9500	4750	2360	1180	600	300	150	75	-75
SS11-MS11	-0.3	4.0	0.2	1.1	-1.3	-0.5	-1.1	-0.6	-0.5	-0.3	-0.7
SS12-MS12	1.1	9.7	-1.2	-2.7	-2.6	-1.6	-0.8	-0.4	-0.3	-0.3	-0.9
SS13-MS13	1.7	9.2	1.4	-3.7	-1.6	-2.3	-1.8	-1.1	-0.5	-0.2	-1.1
SS14-MS14	1.9	-1.6	-2.2	-2.3	0.0	1.0	1.1	0.7	0.4	0.2	0.8
SS15-MS15	2.3	1.4	0.3	-0.1	-1.8	-0.8	-0.3	-0.4	-0.2	-0.2	-0.2
SS16-MS16	1.9	6.2	-0.1	-1.6	-2.6	-0.6	-0.1	-0.4	-0.5	-0.3	-0.1
SS17-MS17	-3.3	-3.2	-0.5	1.8	1.8	1.4	0.7	0.4	0.2	0.2	0.5
SS18-MS18	-1.9	-5.9	2.4	3.4	0.2	1.5	0.5	0.1	0.0	-0.1	-0.2
SS19-MS19	3.9	2.8	0.2	-0.7	-1.2	-1.6	0.4	-0.4	-0.6	-0.3	-1.7
SS20-MS20	2.1	6.3	-1.1	-0.7	-1.9	-1.6	-1.1	-0.6	-0.2	-0.3	-0.9
TOTAL	9.4	28.9	-0.6	-5.5	-11.0	-5.1	-4.2	-2.7	-2.2	-1.6	-5.4

TABLE 17: MEAN DIFFERENCE, STANDARD DEVIATION, STANDARD ERROR, AND CRITICAL RATIO; GROUPS 1-1
(a = 2%, t 0.01; 9 = 2,8214)

DIMENSION (microns)	Mean Diff. d	Standard Dev. Sd	Standard Error Sd	Critical Ratio t
19000	0.19	1.0115	0.3199	0.5940
12500	-0.57	2.3329	0.7377	-0.7726
9500	0.05	1.2616	0.3990	0.1253
4750	0.22	1.0850	0.3431	0.6412
2360	0.24	1.0997	120.3478	0.6901
1180	-0.28	0.8509	0.2691	-0.0406
600	-0.05	0.4972	0.1572	-0.3180
300	0.10	0.3399	0.1075	0.9303
150	0.09	0.2726	0.0862	1.0439
75	-0.03	0.2263	0.0716	-0.4192
UNDERSIZE	0.04	0.5082	0.1607	0.2489

TABLE 18: MEAN DIFFERENCE, STANDARD DEVIATION, STANDARD ERROR, AND CRITICAL RATIO; GROUPS 2-4
(a = 2%, t 0.01; 9 = 2,8214)

DIMENSION (microns)	Mean Diff. d	Standard Dev. Sd	Standard Error Sd	Critical Ratio t
19000	0.94	2.1598	0.6830	1.3763
12500	2.89	5.2361	1.6558	1.7454
9500	-0.06	1.3167	0.4164	-0.1441
4750	-0.55	2.1808	0.6896	-0.7975
2360	-1.10	1.3840	0.43778	-2.5133
1180	-0.51	1.3674	0.4324	-1.1794
600	-0.42	0.9271	0.2932	-1.4325
300	-0.27	0.5271	0.1667	-1.6197
150	-0.22	0.3327	0.1052	-2.0913
75	-0.16	0.2011	0.6366	-2.5159
UNDERSIZE	-0.54	0.7648	0.2418	-2.2328

APPENDIX B

REFERENCES

BAILLAGEON, G. AND RAINVILLE, J., Applied Statistics; Les Editions SME, Trois-Rivieres, 1977.

Article: Standard Practice for Sampling Agregates, Norme D-75; American Society for Testing Materials, 1982.

Article: Standard Recommended Practice for Choice of Sample Size to Estimate the Average Quality of a Lot or Process, Norme E-122, American Society of Testing Materials, 1972.

Article: Sampling Aggregates, Standard 2560-010; Quebec Standards Association, 1982.

Article: Aggregates - Granulometric Analysis by Screening, Standard 2560-040, Quebec Standards Association, 1982.

Article Iron Ores - Increment Sampling Manual Method, Norme 3081; International Organization, 1973.

Article: Coal Sampling, Standard 1988, International Standards Organization, 1978.

Article: Standard Method for Collection of a Gross Sample of Coal, Norme D-2234; American Society for Testing Materials, 1982.

Article: Standard Recommended Practice for Acceptance of Evidence based on the Results of Probability Sampling, Norme E-141; American Society of Testing Materials, 1969.

Article: Standard Recommended Practice for Probability Sampling of Materials Norme E-105, American Society of Testing Materials, 1958.

APPENDIX C

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